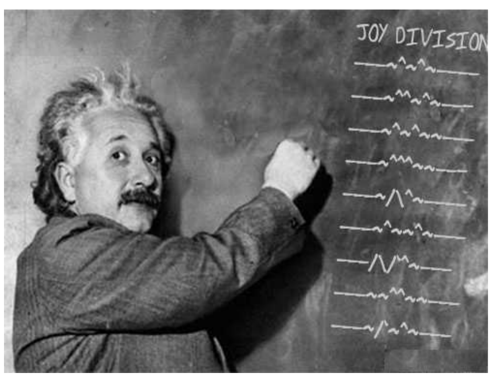
Nuclear Structure from Gamma-Ray Spectroscopy

2019 Postgraduate Lectures

Lecture 9: Electromagnetic Transitions



Electromagnetic Radiation

- The energy of an electromagnetic radiation field can be described mathematically in terms of a multipole moment expansion
- The expansion converges rapidly; hence only the lower orders are of importance
- The terms correspond to 2^n-poles and the lowest terms are named:

$n = 0$	monopole	$n = 1$	dipole
$n = 2$	quadrupole	$n = 3$	octupole
$n = 4$	hexadecapole	…etc	

Why EM Transitions?

- The multipole moments are dependent on charge and current densities in the nucleus and so their study allows information to be extracted on these properties
- Magnetic (M1) moments are sensitive to nuclear magnetic moments and single-particle properties
- Electric (E2) moments are sensitive to the nuclear charge distribution and collective effects such as deformation

Electromagnetic Moments

- The electromagnetic potential due to a finite charge distribution $q(\underline{r}')$ is given by:

$$\Phi(\underline{r}) = (1/4\pi\varepsilon_0) \int q(\underline{r}')d\underline{r}' \,/\, |\underline{r} - \underline{r}'|$$

- For $r > r'$ we can expand:

$$1 / |\underline{r} - \underline{r}'| = 1 / \{r|1 - \underline{r}'/\underline{r}|\}$$
$$= (1/r) \{1 + (\underline{r}'/\underline{r}) + (\underline{r}'/\underline{r})^2 + (\underline{r}'/\underline{r})^3 + (\underline{r}'/\underline{r})^4 + \ldots\}$$

- In terms of spherical harmonics:

$$\Phi(\underline{r}) = (1/4\pi\varepsilon_0)$$
$$\sum_{\lambda\mu} \int \{4\pi q(\underline{r}')(r')^{\lambda}/(2\lambda+1)r^{\lambda+1}\}\, Y'(\theta',\varphi')Y(\theta,\varphi)d\underline{r}'$$

Multipole Expansion

- We can introduce the multipole coefficients:

  $Q_{\lambda\mu} = (1/Z) \int e(r')^{\lambda} Y'_{\lambda\mu}(\theta',\varphi') \rho_{charge}(\underline{r}')d\underline{r}'$

- The potential can then be written as:

  $\Phi(\underline{r}) = (1/4\pi\varepsilon_0) \sum_{\lambda\mu} \{4\pi Z/(2\lambda+1)r^{\lambda+1}\} Q_{\lambda\mu} Y_{\lambda\mu}(\theta,\varphi)$

- Then using $\rho_{charge}(\underline{r}') = |\Psi(\underline{r})|^2$ we can rewrite the multipole coefficients as:

  $Q_{\lambda\mu} = \langle \Psi(r) \mid er^{\lambda} Y'_{\lambda\mu}(\theta,\varphi) \mid \Psi(r) \rangle$

- Multipole moments are tensors of rank λ and parity $(-1)^{\lambda}$ with $2(\lambda+1)$ substates: $-\lambda \leq \mu \leq \lambda$

Electric Multipole Operator

- If we assume that the nuclear wavefunction is made of products of single-particle wavefunctions, then we can write the electric moment operator as:

$$\hat{O}_{\lambda\mu}(E\lambda) = \sum_{protons} e(r_i)^\lambda Y'_{\lambda\mu}(\theta_i,\varphi_i) = \sum_i^A e_i(r_i)^\lambda Y'_{\lambda\mu}(\theta_i,\varphi_i)$$

with $e_i = e$ for protons and $e_i = 0$ for neutrons

- Since $Y_{\lambda\mu}$ has parity $(-1)^\lambda$ all odd-order electric multipole coefficients vanish

- For a spherical nucleus only Q_{00} is nonzero

Magnetic Multipole Operator

- We can define a magnetic charge density as the divergence of magnetization density:

$$\rho_m(\underline{r}') = -\underline{\nabla}.\ \underline{M}(\underline{r})$$

- The magnetization current is:

$$\underline{j}(\underline{r}') = -\underline{\nabla} \times \underline{M}(\underline{r})$$

- The magnetic density multipole coefficient is:

$$M_{\lambda\mu} = \int r^\lambda Y'_{\lambda\mu}(\theta,\varphi)\, \rho_m(\underline{r})\, d\underline{r} = - \int r^\lambda Y'_{\lambda\mu}(\theta,\varphi)\, \underline{\nabla} \times \underline{M}(\underline{r})\, d\underline{r}$$

- Since $M_{\lambda\mu}$ has parity $(-1)^{\lambda+1}$ all even-order magnetic multipole coefficients vanish

Magnetic Multipole Operator

- The magnetic multipole operator is defined as:

$$\hat{O}_{\lambda\mu}(M\lambda) = \mu_N \sum_i^A \{2/(\lambda+1)\, g_{\ell i} \underline{\ell}_i + g_{si} \underline{s}_i\} \,.\, \nabla_i((r_i)^\lambda Y'_{\lambda\mu}(\theta_i,\varphi_i))$$

where μ_N is the nuclear magneton defined as:

$$\mu_N = e\hbar/2m_N c$$

- Recall: Bohr magneton in Atomic Physics which uses the mass of an electron rather than the mass of a nucleon

Transition Matrix Elements

- Consider a transition from a state $|I_1M_1\rangle$ to a state $|I_2M_2\rangle$. The 'matrix element' for the transition is:

$$\langle I_2M_2|\hat{O}_{\lambda\mu}|I_1M_1\rangle$$

- The 'Wigner Eckart Theorem' allows this matrix element to be expressed as:

$$\langle I_2M_2|\hat{O}_{\lambda\mu}|I_1M_1\rangle = (2I_2+1)^{-1/2}\,\langle I_1M_1\lambda\mu|I_2M_2\rangle\,\langle I_2||\hat{O}_{\lambda}||I_1\rangle$$

where $\langle I_2||\hat{O}_{\lambda}||I_1\rangle$ is a 'reduced' matrix element and $\langle I_1M_1\lambda\mu|I_2M_2\rangle$ is a 'Clebsch-Gordon' vector addition coefficient

Reduced Matrix Elements

- Separation of 'orientation' of vectors and 'intrinsic' nuclear properties

- The dependence of the reduced matrix element on the magnetic quantum numbers μ, M_1 and M_2 (i.e. the orientation) is removed

- The reduced matrix element then only contains intrinsic nuclear information

- For EM transitions between states of I_2 and I_1 the following selection rules ensue:

$$M_2 = M_1 + \mu \quad \text{and} \quad |I_2 - I_1| \leq \lambda \leq I_2 + I_1$$

Reduced Transition Probabilities

- The reduced transition probability is defined as:

$$B(O_\lambda; I_1 \rightarrow I_2) = \sum |\langle I_2 M_2 | \hat{O}_{\lambda\mu} | I_1 M_1 \rangle|^2$$

$$= \{1/(2I_1+1)\}\ |\langle I_2 || \hat{O}_\lambda || I_1 \rangle|^2$$

which ensures that the lifetime of a nuclear state does not depend on its orientation (i.e. rotational invariance)

- The relation between the excitation $B(O_\lambda)\uparrow$ and the decay $B(O_\lambda)\downarrow$ of a nuclear state is:

$$B(O_\lambda; I_1 \rightarrow I_2) = \{\ (2I_2+1)/(2I_1+1)\ \}\ \ B(O_\lambda; I_2 \rightarrow I_1)$$

Transition Probabilities

- The transition rate, decays per second, for a specific multipole is given by

$$T(O_\lambda) = \{8\pi(\lambda+1)\}/\{\lambda[(2\lambda+1)!!]^2\}\,\{k^{2\lambda+1}/\hbar\}\,B(O_\lambda)$$

where k is the wave vector of the gamma ray

- Note the strong dependence on k, or gamma-ray energy

- The mean lifetime of a nuclear state is then simply

$$\tau = T^{-1}$$

Transition Rates

- Transition rates (s^{-1}) for the lowest multipoles:

$$T(E1) = 1.590 \times 10^{15}\, E_\gamma^3\, B(E1)$$
$$T(E2) = 1.225 \times 10^{9}\, E_\gamma^5\, B(E2)$$
$$T(E3) = 5.708 \times 10^{2}\, E_\gamma^7\, B(E3)$$
$$T(M1) = 1.758 \times 10^{13}\, E_\gamma^3\, B(M1)$$
$$T(M2) = 1.355 \times 10^{7}\, E_\gamma^5\, B(M2)$$
$$T(M3) = 6.313\, E_\gamma^7\, B(M3)$$

- Units:

E_γ	MeV
$B(E\lambda)$	$e^2\, fm^{2\lambda}$
$B(M\lambda)$	$\mu_N^2\, fm^{2\lambda-2}$

Single-Particle Transitions

- For an electric single-particle transition we assume excitation of only one proton in an average central potential that changes orbit from j_2 to j_1

- A magnetic single-particle transition takes place when the intrinsic spin is flipped, e.g.

$$j_2 = \ell_2 + \tfrac{1}{2} \rightarrow j_1 = \ell_1 - \tfrac{1}{2}$$

- A useful scale of $B(E\lambda)$ and $B(M\lambda)$ values is provided by the Weisskopf single-particle units (W.u) calculated assuming a uniform charge density

Weisskopf Units

- Weisskopf single-particle strengths are:

$$B(E1)_W = 0.06446\ A^{2/3}\ e^2fm^2$$
$$B(E2)_W = 0.05940\ A^{4/3}\ e^2fm^4$$
$$B(E3)_W = 0.05940\ A^{2}\ e^2fm^6$$
$$B(M1)_W = 1.7905\ \mu_N^2$$
$$B(M2)_W = 1.6501\ A^{2/3}\ \mu_N^2fm^2$$
$$B(M3)_W = 1.6501\ A^{4/3}\ \mu_N^2fm^4$$

- Typical experimental values are:

$B(E1) \sim 10^{-2}$ W.u. ; $B(M1) \sim 10^{-1}$ W.u. ; $B(E2) \sim 10^{2}$ W.u.

Single-Particle Transition Rates

- Using the Weisskopf estimates for reduced transition probabilities the following single-particle transition rates are found:

E1 $T_{sp} = 1.025 \times 10^{14} E_\gamma^3 A^{2/3}$ s^{-1}

E2 $T_{sp} = 7.276 \times 10^{7} E_\gamma^5 A^{4/3}$ s^{-1}

E3 $T_{sp} = 3.339 \times 10^{1} E_\gamma^7 A^{2}$ s^{-1}

M1 $T_{sp} = 3.148 \times 10^{13} E_\gamma^3$ s^{-1}

M2 $T_{sp} = 2.236 \times 10^{7} E_\gamma^5 A^{2/3}$ s^{-1}

M3 $T_{sp} = 1.042 \times 10^{1} E_\gamma^7 A^{4/3}$ s^{-1}

- Note: low multipolarities are favoured. Electric transitions are faster than magnetic transitions

Magnetic Dipole Moment

- The magnetic dipole moment $\underline{\mu}$ provides a measure of the current distribution in a nucleus. It is generated by the orbital motion of the protons (current loop) and the intrinsic spins of all nucleons

- The magnetic dipole moment operator is:

$$\underline{\mu} = \mu_N \sum_1^A \{g_{\ell i} \underline{\ell}_i + g_{si} \underline{s}_i\}$$

- The orbital and spin g-factors for free nucleons are:

proton: $g_\ell = 1, \quad g_s = 5.5856$

neutron: $g_\ell = 0, \quad g_s = -3.8262$

Effect of the Core

- Single particle g-factors are usually denoted g_K

- A core contribution to the magnetic moment can be estimated by assuming the protons are evenly distributed throughout the nucleus which is rotating with core angular momentum $\underline{R}$:

$$\underline{\mu} = g_R \underline{R} \mu_N \qquad \text{with} \qquad g_R \approx Z/A$$

- Since $\underline{I} = \underline{R} + \underline{j}$, the magnitude of μ can be written as:

$$\mu = g_R I + (g_K - g_R) \{K^2/(I+1)\}$$

Reduced M1 Transition Rate

- The reduced matrix element of the magnetic dipole moment operator leads to the following expression for the reduced M1 transition rate (units μ_N^2):

$$B(M1; I \rightarrow I-1) = \{3/4\pi\} (g_K - g_R)^2 K^2$$

$$\times \{1 + (-1)^{I+1/2} b\} |\langle I\ K\ 1\ 0 | I-1\ K \rangle|^2$$

where $\langle I\ K\ 1\ 0 | I-1\ K \rangle$ is a Clebsch-Gordon vector addition coefficient

- The quantity b is the magnetic decoupling parameter and is only nonzero for bands with $K=1/2$

Electric Quadrupole Moment

- The electric quadrupole moment Q_0 (strictly $Q_{2\mu}$ or Q_{20} for axially symmetric shapes) provides a measure of the charge distribution of the nucleus

- The corresponding electric quadrupole operator is:

$$e\,\underline{Q}(r) = \int \rho(\underline{r})\,(3\cos^2\theta - 1)\,dV$$

- The intrinsic quadrupole moment is defined as the expectation value of this operator $\underline{Q}(r)$ for a nucleus in the state $|I,M\rangle$:

$$Q_0 = \langle I,M|\underline{Q}(r)|I,M\rangle$$

Spectroscopic Quad. Moment

- The intrinsic quadrupole moment Q_0 is defined in the nuclear frame of reference.

- The spectroscopic quadrupole moment Q_S is defined in the laboratory frame:

$$Q_S = \langle I, M=I | \underline{Q}(r) | I, M=I \rangle$$

where the state $|I,M=I\rangle$ defines Q_S as the maximum observable quadrupole moment

- These quantities are related by:

$$Q_S = Q_0 \{3K^2 - I(I+1)\} / \{(I+1)(2I+3)\}$$

Reduced E2 Transition Rate

- The reduced matrix element of the electric quadrupole operator leads to the following expression for the reduced E2 transition rate (e^2b^2):

$$B(E2; I \rightarrow I-2) = \{5/16\pi\}\, Q_0^2 |\langle I\ K\ 2\ 0 | I-2\ K \rangle|^2$$

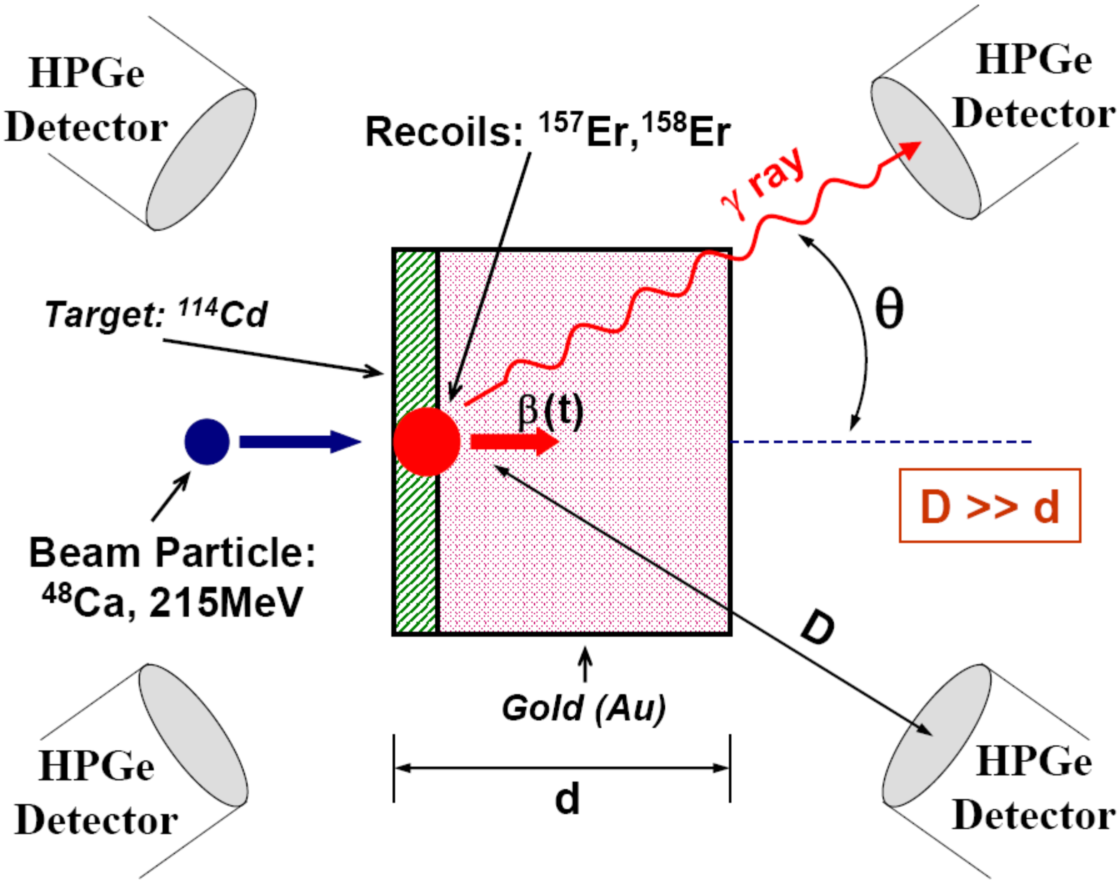
where $\langle I\ K\ 2\ 0 | I-2\ K \rangle$ is a Clebsch-Gordon vector addition coefficient

- The mean lifetime of a state decaying by a stretched E2 transition is:

$$\tau(\text{ps}) = 0.0816 \,/\, \{\, E_\gamma^5\,(\text{MeV})\; B(E2)\,(e^2b^2)\,\}$$

Quadrupole Moments

- A DSAM lifetime measurement (12 days) was carried out at the ATLAS facility at ANL using Gammasphere (~100 HPGe)
- Fractional Doppler shifts F were measured

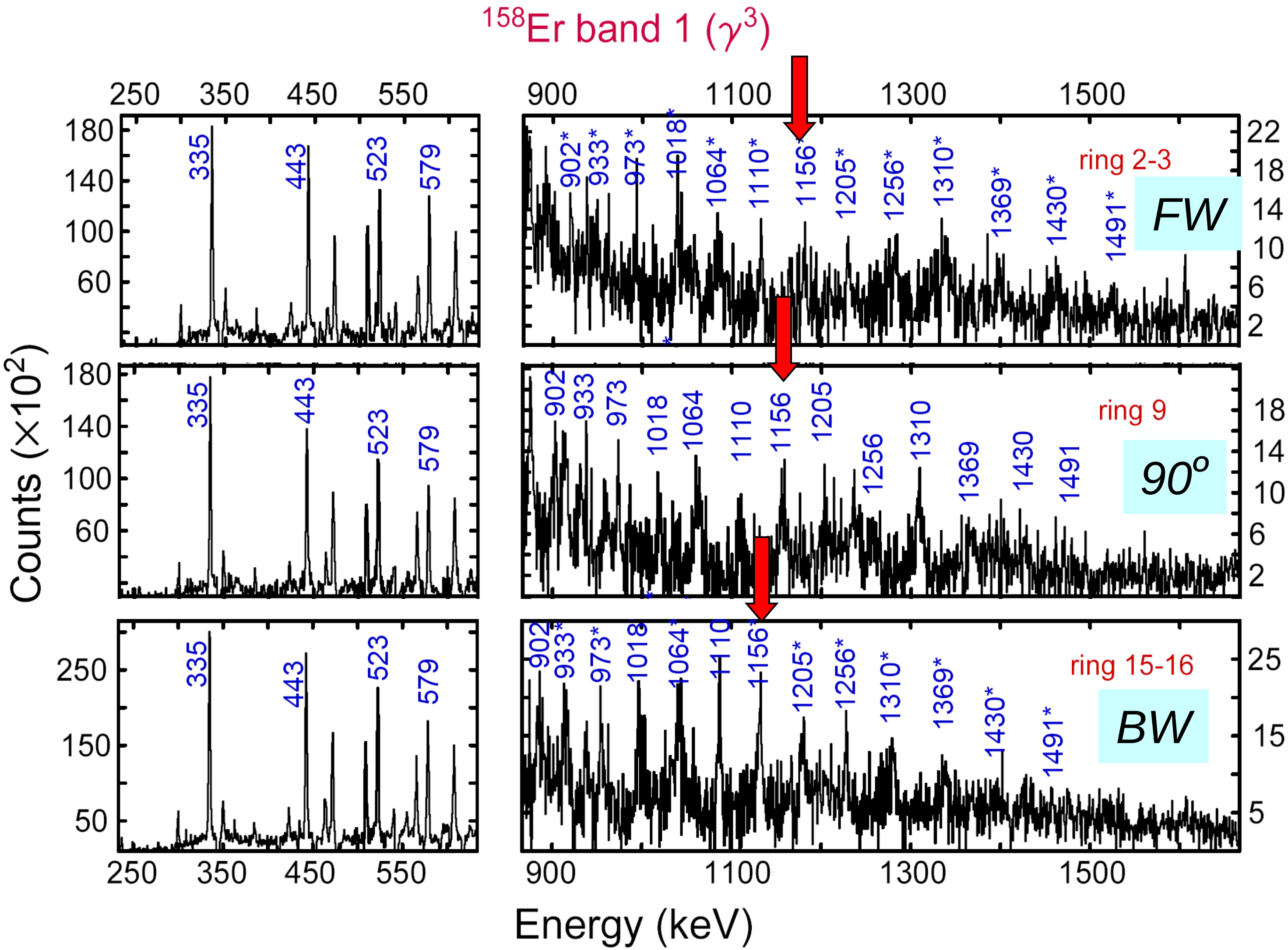


$F = \beta(t)/\beta_0$

$\beta = v/c$

$E(\theta) = E_0(1+F\beta_0\cos\theta)$

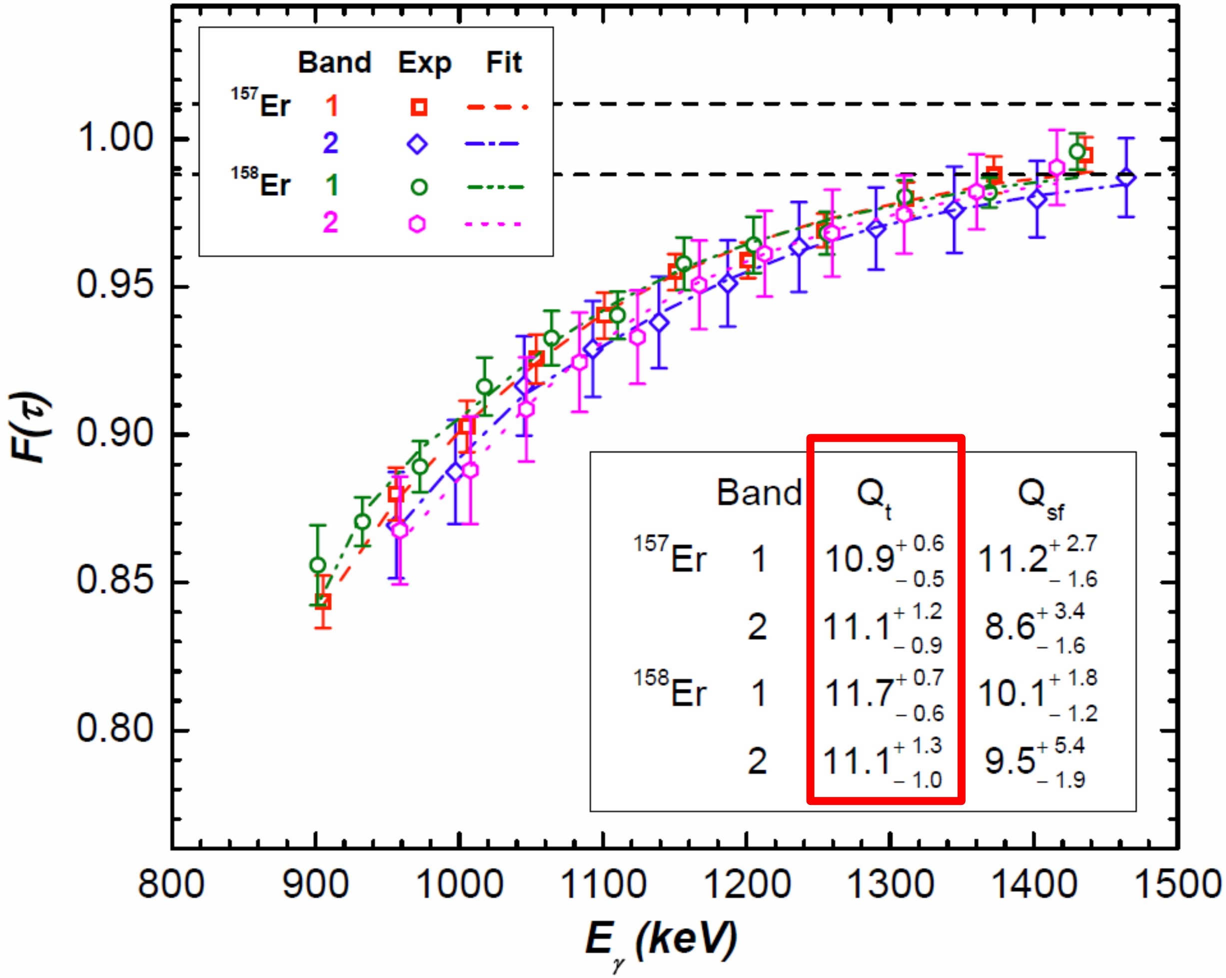
Lifetime Measurements in ^{158}Er



➢ Gammasphere experiment GSFMA229

^{157,158}Er Quadrupole Moments

- Gammasphere experiment GSFMA229



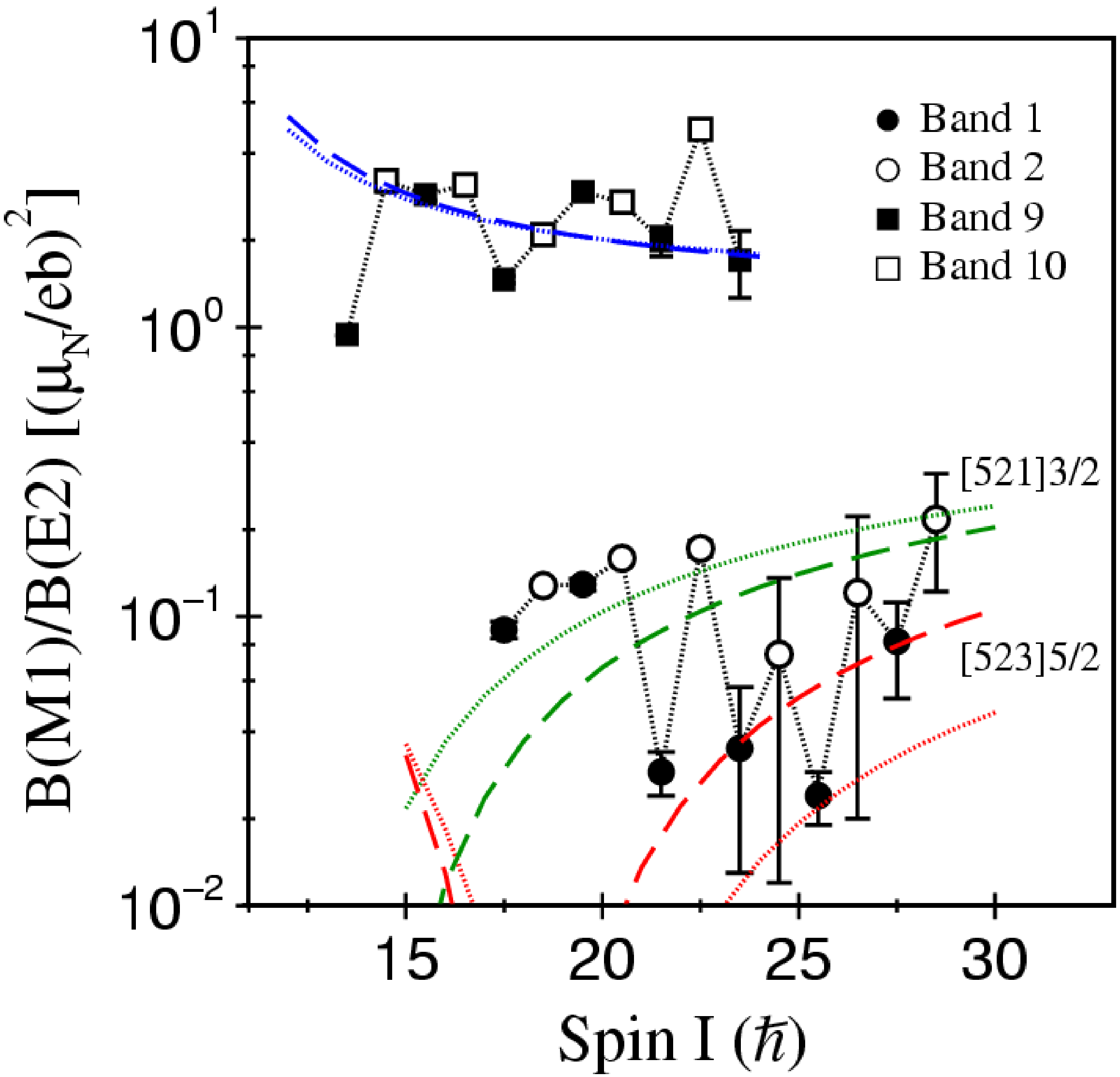
Band		Q_t	Q_{sf}
^{157}Er	1	$10.9^{+0.6}_{-0.5}$	$11.2^{+2.7}_{-1.6}$
	2	$11.1^{+1.2}_{-0.9}$	$8.6^{+3.4}_{-1.6}$
^{158}Er	1	$11.7^{+0.7}_{-0.6}$	$10.1^{+1.8}_{-1.2}$
	2	$11.1^{+1.3}_{-1.0}$	$9.5^{+5.4}_{-1.9}$

Spin range covered: 33 – 57ħ

B(M1)/B(E2) Ratios

- Experimentally it is difficult to obtain absolute B(M1) and B(E2) values through measurements of the mean lifetimes of nuclear states
- In contrast, it is relatively easy to extract the ratio B(M1)/B(E2) knowing just γ-ray energies and intensities
- The ratios are very sensitive to nuclear configurations in strongly coupled (high K) bands
- Donau and Frauendorf geometric model

B(M1)/B(E2) ratios in ^{157}Er



- B(M1)/B(E2) ratios for bands in ^{157}Er
- B(M1) is sensitive to the single-particle configuration :
  g-factor
- B(E2) is sensitive to the collectivity:
  quadrupole moment